A Reference Dependent Regret Theory

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Abstract

We propose an extension of the classical regret theory model (Loomes & Sugden, 1982; henceforth LS) incorporating the notion of a reference point. As in LS, the model can account for a number of documented deviations from expected utility theory. Additionally, we show that our model is consistent with a class of behaviors known as *omission bias*, for example a reluctance to exchange lottery tickets, and generates predictions which are consistent with recent empirical evidence on the common-ratio effect with correlated outcomes. The model also provides a novel interpretation for risk aversion in small-stakes, equiprobable gambles. The predictive power of the theory, as well as its relative shortcomings and advantages, are examined and compared to that of other extensions of regret theory and three alternative reference-dependent models.

Keywords: Regret Theory, Reference Point, Risky Choice, Omission Bias, Lottery Exchange

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Introduction

Regret theory (Loomes & Sugden, 1982, henceforth LS) is a modification of the expected utility (EU) model which incorporates the notion that decision makers care not only about the realized consequences of their choices, but also about counterfactual outcomes that would have obtained had they chosen differently.[1] In LS's model, a decision maker chooses between two plans of action (i.e., acts) by comparing the mathematical expectation of their *modified* utilities. For each possible state of the world, the utility of a particular act is a function of the outcome obtained in that state — as in the canonical EU model — *and* of the difference in the utilities had the other course of action been chosen instead.

Regret theory has been successful in explaining behaviors inconsistent with the standard EU model across a variety of decision-making domains. Barberis, Huang, and Thaler (2006), for instance, use the concept of regret as an explanation for the stock market participation puzzle, i.e., the observed reluctance to invest in the stock market despite it being an actuarially favorable gamble. Muermann and Volkman Wise (2006) develop a model of regret which is consistent with the disposition effect, namely, the tendency of investors to hold on to losing assets and to more quickly realize gains. In term of insurance choices, Braun and Muermann (2004) use a regret model to explain the observed preferences for low-deductible policies. There are also numerous studies on the importance of regret in the context of health-related decisions (Djulbegovic, Hozo, Schwartz, & McMasters, 1999; Richard, 1994; Weinstein, 1986).[2]

Since regret is a counterfactual emotion, its intensity is highly context-dependent (Kahneman & Tversky, 1981), and can even vary within the same decision problem. For example, one is more likely to experience an anticipated feeling of regret when less mental effort is involved in visualizing the outcome of a forgone alternative (Harless, 1992). Furthermore, the feeling of regret associated with a poor outcome is less intense when brought upon by a "passive" choice (or inaction) rather than an "active" choice (Kahneman & Tversky, 1982). Put differently, the same unfavorable outcome is deemed

[1] Fishburn (1982) and Bell (1982) are other seminal contributions to regret theory. We will focus on the model in Loomes and Sugden (1982), for it is most closely related to the psychological underpinnings of regret and rejoicing.

[2] See Bleichrodt and Wakker (2015), Diecidue and Somasundaram (2017), Somasundaram and Diecidue (2017) and Zeelenberg (2018) for an extensive list of references on the applications of regret theory.

more aversive if perceived as resulting from bad choices rather than from bad luck, leading to a type of behavior known in the literature as *omission bias* (Baron & Ritov, 1994; Jamison, Yay, & Feldman, 2020; Ritov & Baron, 1992).[3]

Despite their relevance to the psychology of regret, such factors have yet to be incorporated in theoretical models of regret aversion. Consequently, in some domains of decision making under risk, LS's model is unable to account for certain behaviors even though a regret motive appears to be involved in the decision process. A prominent example is the omission bias mentioned earlier. It has been shown, for example, that some parents might prefer to refrain from vaccinating their children (i.e., a bias towards inaction) even if the risk of a deadly outcome from the disease is higher than the risk from the vaccine (Asch et al., 1994; Ritov & Baron, 1990, 1992). There is also evidence that people are reluctant to exchange lottery tickets — an act that bears no implications on the objective probabilities — even when a small monetary incentive is offered in return (Bar-Hillel & Neter, 1996; Van de Ven & Zeelenberg, 2011). Additionally, the common-ratio (or certainty) effect, which is a violation of the standard EU model, is observed even when using correlated, state-contingent payoff tables, which facilitates the incorporation of counterfactual-based regret and, according to LS's model, should moderate this effect (Loomes & Sugden, 1998). Lastly, for a particular set of decision problems under risk (a gamble with 50-50 chance of winning or losing different monetary amounts), regret theory generates the same predictions as the EU model, which renders it susceptible to Rabin's critique: risk aversion for actuarially favorable small-stakes gambles results in an implausible amount of risk aversion for high-stakes gambles (Rabin, 2000; see also Rabin & Thaler, 2001).

In this paper we propose a *reference dependent regret theory* — henceforth RDRT — as an extension of LS's regret theory that is able to accommodate the phenomena described earlier. The model incorporates the notion of a reference point into the original formulation of LS to allow for asymmetric feelings of regret and rejoicing. It rests upon the idea that, following an unfavorable realization, an "active" choice — i.e., a choice that leads to an outcome *far away* from the reference point — generates a relatively stronger sense of regret than a "passive" choice — i.e., a choice that leads to

[3] Barberis et al. (2006) make a similar point using an example of real-life financial decision making: "Regret is thought to be stronger, however, when it stems from having taken an action, for example, moving one's savings from the default option of a riskless bank account to the stock market, than from not having taken an action, for example leaving one's savings in place at the bank." (footnote 15, p. 1084).

an outcome *closer* to the reference point. We show that RDRT is consistent with (1) the omission bias, such as the reluctance to exchange lottery tickets, and (2) the common-ratio effect with juxtaposed presentation. Moreover, the model provides an interpretation of Rabin's critique that does not imply implausibly large risk aversion levels for high-stakes, actuarially fair gambles.

The rest of the paper is organized as follows. We begin by providing an overview of LS's regret theory and discussing some examples that motivate our new theory. We next introduce the RDRT model and provide a number of comparisons to alternative models in the literature. We conclude with a general discussion of the theory and possible future directions.

LS's Model and Motivating Examples

A Brief Introduction to Loomes and Sugden (1982)

To capture the effect of regret in the decision-making process, LS define an environment where elements of the choice set are acts with state-contingent consequences, as opposed to prospects with probability distributions defined over outcomes. This modeling choice leads naturally to a measure of regret (or rejoicing) via state-by-state comparisons between the outcomes of any two acts. Formally, let S be a finite set of possible states of the world with probability measure P on S. Each state is denoted by s. In addition, let $u(\cdot)$ be an utility function which is unique up to an affine transformation. LS refer to $u(\cdot)$ as a "choiceless" utility function: the well-being an individual experiences from an outcome without having chosen it. This is similar to Bernoulli's original concept of utility as a psychological phenomenon (Stigler, 1950); for this reason, we shall refer to $u(\cdot)$ as a Bernoulli utility index.

Feelings of regret or rejoicing are captured by a real-valued function $\Phi(.)$ — the *regret-rejoice* function — which maps differences in well-being from any two outcomes into the real line. For any two acts x and y, and any state of the world s, $\Phi\left(u(x_s) - u(y_s)\right)$ encodes the feeling of regret or rejoicing from choosing x over y, where x_s (y_s) is the outcome obtained in state s by choosing x (y). It is natural to assume that $\Phi(0) = 0$, which means that the decision maker experiences neither regret nor rejoicing when state s occurs whenever both acts have the same consequence in state s. The function $\Phi(\cdot)$ is further assumed to be strictly increasing and three times differentiable.

Suppose a decision maker is facing a choice set with two alternative acts x and y. According to regret theory, the utility from choosing x is equal to the expected value of the "choiceless utility" and the regret components, which LS refer to as *modified utility*:

$$\sum_{s\in S} p_s\left[u(x_s)+\Phi\left(u(x_s)-u(y_s)\right)\right]$$

Let

$$Q(\xi)=\xi+\Phi(\xi)-\Phi(-\xi)$$

and note that $Q(\cdot)$ is increasing in ξ, with $Q(0)=0$ and $Q(-\xi)=-Q(\xi)$. For any two arbitrary acts, x and y, preferences can be represented as follows:

$$x\succeq y\Longleftrightarrow\sum_{s\in S} p_s Q\left(u(x_s)-u(y_s)\right)\geq 0 \tag{1}$$

If the function $Q(\cdot)$ is linear, (1) is equivalent to the EU theory representation. However, the most common assumption about $Q(\cdot)$ in the regret literature is that for any set of outcomes x, y, z with $u(x)>u(y)>u(z)$,

$$Q(u(x)-u(z))>Q(u(x)-u(y))+Q(u(y)-u(z))$$

which is known as the property of convexity.[4] Combining the convexity of $Q(\xi)$, for $\xi>0$, with a monotonically increasing utility function $u(\cdot)$, regret theory is able to predict a number of empirically validated deviations from EU. Specifically, LS show that, based only on these two conditions (and treating prospects as statistically independent), regret theory predicts (1) the "common-ratio" (or "certainty") effect, (2) the "common-consequence" effect (or Allais' Paradox), and (3) the "isolation effect", which is a violation of the property of reduction of compound lotteries. With additional constraints, regret theory is also consistent with the "reflection" effect — risk aversion in the gain domain and risk loving the loss domain — and with simultaneous gambling and insurance.[5]

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[4] This property, also referred to by LS as regret aversion, entails that for all $\xi>0$, $Q(\xi)$ is convex. This, in turn, implies that $Q(\xi)$ is concave for all $\xi<0$.

[5] These results obtain either with (i) a linear utility function or (ii) with a concave utility function coupled with restrictions on the probability distribution.

A key insight from regret theory is that the state at which a consequence is obtained is of key importance to the decision-making process. Consider, for instance, the decision problem presented in Table 1, with four equiprobable states of the world (s_1, s_2, s_3, s_4). Note that acts x and y result in identical distributions over final outcomes — 25% chance of each of \$0, \$10, \$20 or \$30. If analyzed as prospects, rather than state-dependent acts, x and y are equivalent. As such, both EU theory and other any prospect-based model, would predict indifference between x and y.

Let us now analyze the choices under regret theory. The preference relation is:

$$x \succeq y \iff \sum_{s=1}^{4} p_s Q(u(x_s) - u(y_s)) \geq 0$$

Assume, for simplicity, that $u(x) = x$.[6] We thus obtain:

$$x \succeq y \iff 0.75 \times Q(10) - 0.25 \times Q(30) \geq 0 \tag{2}$$

which, owing to the convexity of $Q(\cdot)$, result in the strict preference $y \succ x$. Intuitively, a regret-averse decision maker would prefer a relative loss of \$10 in states s_1, s_2 or s_3 to a larger relative loss of \$30 in state s_4.

An important implication of regret theory is that choices need not be transitive. Consider the example in Table 2, which extends Table 1 by adding two new acts z and w. The four acts have identical probability distributions over outcomes. Let us again assume, for simplicity, that $u(x) = x$. As shown in (2), $y \succ x$ due to the convexity of $Q(\cdot)$. For the same reason, we also have that $x \succ w$, $w \succ z$, and $z \succ y$, which creates a cycle of preferences that violates transitivity. We should note that LS provide both positive and normative arguments in support of this implication, which we generally agree with. We shall return to these points in the discussion section.

Empirical Evidence and Limitations

Testing the empirical validity of regret theory was the natural next step after the theory was introduced. Though many empirical studies provide supporting evidence (see Bleichrodt and Wakker (2015) for a review), contradictory findings are also present in the literature. Harless (1992), for instance, argues that the effect of regret hinges

[6] This assumption is made for ease of exposition. The same results are achieved with a concave utility function, as long as the convexity of $Q(\cdot)$ is *large enough*.

heavily on the way decision problems are presented. He shows that regret theory is mostly consistent with a matrix representation that requires virtually no mental effort in visualizing the state-by-state implications of forgoing an alternative. A more "ambiguous" presentation, such as verbal presentations or proportional formats, results in a much weaker (or even null) regret effect. Further contradicting evidence is provided by Starmer and Sugden (1993), who argue that results from previous studies were moderated by an "event-splitting" effect. According to this effect, the weight placed on an event with a given consequence is subadditive, i.e., splitting the event into multiple sub-events increases its perceived weight.

Importantly, the two aforementioned papers do not attempt to invalidate the underlying claim that anticipated feelings of regret or rejoicing affect choice behavior. Their goal, instead, is to reevaluate the validity of results reported in previous studies, especially those that use matrix-based presentations with juxtapositions. We take a similar approach in this paper, maintaining the psychological phenomenon of regret aversion while focusing on limitations of the regret theory model itself. We now turn to a discussion of such limitations using three motivating examples.

Lottery Exchange. Consider the following scenario. You are given the option to choose between two doors, x and y. Behind one door rests a monetary prize of one million dollars, while nothing lies behind the other door. The location of the prize has been previously determined by a flip of a fair coin. Suppose you have chosen door x and have been told to wait one hour before opening the doors. After 59 minutes of waiting, you are asked to make another choice: keep your original decision (i.e., door x) or switch to door y. Would you be willing to switch? Clearly, the chances to win the prize remain the same (50%). However, we suggest that the feeling of regret would be *stronger* if the zero-dollar outcome were to occur following a decision to switch to door y compared to staying with door x. Envisioning such feelings of regret might lead you to strictly prefer to stay with door x. We shall refer to this scenario as one of "bad luck" versus "bad choice", where bad luck occurs when you do not win the money after keeping with the status quo (door x), while a bad choice happens when you do not win the prize after making an active choice (switching to door y).

Examples of similar behaviors are found in a number of studies (Bar-Hillel & Neter, 1996; Kogler, Kühberger, & Gilhofer, 2013; Van de Ven & Zeelenberg, 2011). Bar-Hillel and Neter (1996), for example, conducted an experiment in which students received lottery tickets, each with the same probability of winning, and were then offered a small monetary incentive to exchange their ticket for another one. More than 50% of the students turned down the offer, with some treatments reporting over 70% refusal. The authors then applied the same procedure using pens instead of lottery tickets, yielding significantly different results: less than 10% of students were reluctant to exchange their pens. This suggests that regret rather than a pure endowment effect was the source of the exchange aversion.[7]

The Common Ratio Effect with Juxtaposition. In their seminal paper, Kahneman and Tversky (1979) report on a series of (hypothetical) decisions between pairs of prospects. In one such decision, ninety five subjects were asked to choose between a certain amount of 3,000 pounds or a lottery that pays 4,000 with 80% chance (0 otherwise). Seventy six participants (80%) chose the certain amount. The same participants were then offered a choice between two lottery tickets: 3,000 with 25% chance or 4,000 with 20% chance. In this case, 65% preferred 4,000 with 20% chance. Such preferences are inconsistent with EU, as the latter options are obtained by simply multiplying the initial prospects by 25%. This pattern of preferences is known as the *certainty* or *common-ratio* effect.

As previously mentioned, regret theory can account for the common ratio effect under general assumptions — convex $Q(\cdot)$ and monotonically increasing $u(\cdot)$ — provided the relevant acts are generated from *statistically independent* prospects. When the above-mentioned decision problems are displayed in a correlated way using juxtaposition, however, regret theory predicts the nullification of the common-ratio effect. Table 3 illustrates this assertion.

Let us again assume, for simplicity, that $u(x) = x$.[8] From Panel A of Table 3, a regret-averse decision maker prefers x over y if and only if:

$$\sum_{s=1}^{2} p_s Q(u(x_s) - (y_s)) = 0.2Q(3,000) + 0.8Q(-1,000) \geq 0 \tag{3}$$

Note that a decision maker that prefers x over y would also choose x' over y'. To see this, let us write the modified utility from choosing x' over y' — Panel B of Table 3 —, again assuming that $u(x) = x$:

[7] See also Kogler et al. (2013) for a discussion of endowment and regret effects and Risen and Gilovich (2007) for a discussion of regret effects and perceptions of objective probabilities.

[8] As before, this assumption is used for ease of exposition and is not necessary for the result.

$$x' \succeq y' \iff \sum_{s=1}^{2} p_s Q(u(x_s) - (y_s)) = 0.05Q(3,000) + 0.2Q(-1,000) \geq 0 \tag{4}$$

which is greater than or equal to 0 whenever (3) holds.

Is there empirical support for the common ratio effect when choice alternatives are juxtaposed? Loomes and Sugden (1998) provide evidence in this direction, weakening a central prediction of regret theory. In their experiment, participants (N = 46) responded to a series of 45 binary choice problems in two rounds — the paper's primary goal was to study the stochastic nature of choice behavior. Each binary choice was taken from a Marschak-Machina triangle and presented to subjects as two correlated acts in a particular way as to control for the event-splitting effect. Figure 1 provides an example of the problem visualization, whereas Table 4 presents four decision problems as acts in the usual matrix format.

Note that options x' and y' are proportional to options x and y, respectively, by a factor of 1/4; and options z' and w' are proportional to z and w by a factor of 1/3. As discussed previously, regret theory as in LS would predict that decisions makers who choose x over y should also choose x' over y' — similarly for z, w, z' and w'. In Loomes and Sugden (1998), however, the majority of respondents (twenty nine) chose x over y in both rounds, while twenty two of them also chose y' over x' in both rounds. Similar patterns were also recorded for the third and fourth decision problems. These results lend support to the common ratio effect with correlated acts presented with juxtaposition, but runs contrary to the predictions of regret theory.

Regret Premium with Equiprobable Outcomes. Regret theory can accommodate various deviations from EU-based behavior in a tractable way. The model's parsimony is partly obtained by the symmetric treatment of regret. That is, any given combination of outcome and counterfactual yields the same measure of regret (or rejoicing) irrespective of the chosen alternative. However, in a narrow, yet important, class of choice problems under risk, this built-in symmetry generates predictions which coincide with the standard EU model, as the following proposition suggests.

Proposition 1 *Let x be an arbitrary act with two equiprobable consequences x_1 and x_2. Let z be a constant act such that $z_1 = z_2 = z$. Then, for any monotonically increasing regret-rejoice function $\Phi(\cdot)$,*

$$u(z) \geq \frac{1}{2}\sum_{s=1}^{2} u(x_s) \Longleftrightarrow \frac{1}{2}\sum_{s=1}^{2} Q(u(z) - u(x_s)) \geq 0$$
$$u(z) \leq \frac{1}{2}\sum_{s=1}^{2} u(x_s) \Longleftrightarrow \frac{1}{2}\sum_{s=1}^{2} Q(u(z) - u(x_s)) \leq 0$$

Proposition 1 states that if z is an EU certainty equivalent for an act with two equiprobable outcomes, it is also the certainty equivalent under the regret theory model.

Following Diecidue and Somasundaram (2017), we decompose the risk premium under regret into regret premium and risk premium under expected utility, as follows:

$$\begin{aligned} \textit{Risk Premium} &= \textit{Regret Premium} + \textit{Risk Premium under EU} \\ &= (CE_{EU} - CE_{RT}) + (EV - CE_{EU}) \end{aligned} \tag{5}$$

The first component is the regret premium, i.e. the extra amount that a decision maker pays to avoid regret as compared to an EU maximizer. Under LS's regret theory, the regret premium is determined by the convexity of $Q(\xi)$ and the concavity of $u(\cdot)$. The second component is the risk premium under expected utility. The concavity of the Bernoulli utility function $u(\cdot)$ determines the risk premium under EU. Proposition 1 suggests that the regret premium is zero with equiprobable outcomes, such that a regret-averse decision maker will behave just like an EU maximizer when faced with such lotteries.

The immediate implication is that, for this class of choice problems, regret theory fails to capture the reflection effect, by which risk-averse preferences prevail for prospects with positive payoffs, while risk seeking behavior is observed for prospects involving losses of the same magnitudes. Furthermore, it exposes regret theory to Rabin's critique; i.e. with essentially any (continuously differentiable) concave utility function, the existence of first-order risk aversion for small-stakes gambles amounts to implausible levels of risk aversion for large gambles (Rabin, 2000). Tversky and Kahneman (1991), for instance, found in one of their many experiments that "a 50-50 bet to win \$25 or lose \$10 USD was barely acceptable." (Tversky & Kahneman, 1991, p. 2). This naturally implies a rejection (at least on average) of a 50/50 win \$11 or lose \$10 gamble. If this rejection holds for any level of initial wealth, then, according to EU theory (and standard regret theory), it implies the reluctance to accept a prospect with a 50/50 chance of losing \$100 or gaining any sum of money due to the very high curvature of the utility function.[9] Bleichrodt et al. (2019) shows experimentally that

[9] The rejection of the gamble over a fairly small range of wealth levels is already sufficient to force

Rabin's critique underscores the importance of reference dependence which, as detailed later in the paper, is our key improvement upon original regret theory.

The examples discussed above emphasize some of the main shortcomings of regret theory in its current form. We shall now present a reference-dependent regret model that is able to address these issues. After introducing the model, we will discuss the extent to which three alternative generalized EU models can also overcome these shortcomings and compare their predictive power with that of RDRT.

A Reference-Dependent Regret Model

In LS's regret model, choices are determined by the state-by-state utility differences between any two acts. That is, the measure of regret (or rejoicing) for a fixed util loss (or gain) is independent of the chosen alternative. It is likely, however, that a stronger feeling of regret is associated with choices that are perceived as "more active" (Kahneman & Tversky, 1982). In particular, such asymmetry in regret may be determined by the existence of a reference point: A choice will be considered more active the *farther* away the consequences of a given act are from the reference point. In what follows, we extend LS's regret theory to incorporate the notion of a reference point, thus allowing for asymmetric feelings of regret and rejoicing.

Let r be a state-contingent reference point, which may or may not be deterministic. While many different factors can shape r, the focus in this paper is not on what determines the reference point but rather on how it affects the degree of regret. Hence, at this point we assume r is fixed and equal to the most recent beliefs regarding state-contingent wealth. We now turn to formulating a measure of a choice's relative degree of *activeness* using a weighted Euclidean distance function. For any arbitrary act x, the weighted distance between x and r is:

$$\tilde{\rho}(x,r) = \sqrt{\sum_{s\in S} p_s \left[u(x_s) - u(r_s)\right]^2}$$

where $u(\cdot)$ is a Bernoulli utility function. For any two acts x and y, we capture the weight an individual places on the feelings of regret and rejoicing from choosing x over y by the relative distances of the two alternatives. We refer to it as the regret-weighting index and denote it by $\rho_r(x|y)$:

unrealistic levels of risk-aversion. Hence, while a rejection at any level of initial wealth makes the paradox more conspicuous, it is not necessary.

$$\rho_r(x|y) = \frac{\tilde{\rho}(x,r)}{[\tilde{\rho}(x,r) + \tilde{\rho}(y,r)]/2}$$

It is straightforward to show that $\rho_r \in [0,2]$. With the regret-weighting index, the preference relation (1) becomes:

$$x \succeq_r y \Longleftrightarrow \sum_{s\in S} p_s [u(x_s) - u(y_s) + \rho_r(x|y)\Phi(u(x_s) - u(y_s)) - \rho_r(y|x)\Phi(u(y_s) - u(x_s))] \geq 0$$

Define $Q_r(\xi) = \xi + \rho_r\Phi(\xi) - (2-\rho_r)\Phi(-\xi)$, where ξ is a real number and $\rho_r \in [0,2]$. Letting $\xi = u(x_s) - u(y_s)$ and $\rho_r = \rho_r(x|y)$, we obtain:

$$x \succeq_r y \Longleftrightarrow \sum_{s\in S} p_s Q_r(u(x_s) - u(y_s)) \geq 0 \tag{6}$$

The introduction of the weighting index ρ_r relaxes the symmetric treatment of utility differences of any two acts in a given state of the world. In the special case where x and y are symmetric with respect to r, we have that $\rho_r(x|y) = \rho_r(y|x) = 1$, which implies that the model reduces to LS's regret theory. This will, however, not be the case in general as long as the decision maker cares about a reference point, as our upcoming discussion will illustrate. Note also that if one of the alternatives is the reference point, for instance $x = r$, then $\rho_r(x|y) = 0$, and we arrive at a limiting case in which the individual experiences no regret or rejoicing from choosing x (maintaining the status quo) over y, whereas she places the full weight of regret feelings on the alternative y — i.e., $\rho_r(y|x) = 2$.

We shall assume the following properties:

A1 For all r, $Q_r(\xi)$ is continuous, strictly increasing, three times differentiable in ξ, and $Q_r(0) = 0$. Equivalently, $\Phi(\xi)$ is continuous, strictly increasing, three times differentiable in ξ, and $\Phi(0) = 0$.

A2 For all r, $Q_r(\xi)$ is convex for all $\xi > 0$. Equivalently, fixing ρ_r, $\rho_r\Phi''(\xi) > (2-\rho_r)\Phi''(-\xi)$ for all $\xi > 0$.

A3 $\Phi'(\xi) < \Phi'(-\xi)$ for all $\xi > 0$.

Properties A1 and A2 determine the structure of the regret-rejoice function and give rise to regret aversion as defined in LS. These properties are simple extensions to the reference-dependent context of those originally proposed by LS. Property A3 is novel and unique to RDRT. It is closer in spirit to the notion of loss aversion, or, in this context, loss-regret aversion: Loss-regret weighs more than gain-rejoice. The idea is formalized in the following lemma:

Lemma 1 *Suppose properties A1 and A3 hold. Then for any* $\xi_1 > \xi_2 \geq 0$,

$$\Phi(\xi_1) + \Phi(-\xi_1) < \Phi(\xi_2) + \Phi(-\xi_2).$$

Lemma 1 says that a *loss* of one util due to a foregone alternative generates a feeling of regret that is stronger (in absolute terms) than the rejoice experienced from a *gain* of one util.[10] In addition, given any state of the world, Lemma 1 implies that the feeling of rejoicing is less sensitive to increments in utility differences than the feeling of regret; loosely speaking, an increase of $100 to a favorable (chosen) outcome affects the feeling of rejoicing less than it does the feeling of regret if the alternative was chosen.

The introduction of a reference point crucially affects the preference relations over pairwise alternatives. A shift in the location of the reference point changes the values of the regret-weighting indices. Such changes affect preferences and might cause the individual to reverse her original choice. This brings one immediate concern to mind: could RDRT result in non-monotonic behavior? That is, is it possible to obtain an indifference relation between two alternatives, x and y, for multiple values of the regret-weighting index ρ_r? Proposition 2 shows that such behavior is ruled out by the model.

Proposition 2 *Suppose properties A1-A3 hold and let* x *and* y *be two arbitrary acts such that* $x \neq y$*. If* $\rho_{\hat{r}}$*, for an arbitrary* $\hat{r}$*, is such that* $x \sim_{\hat{r}} y$*, then* $x \succ_r y$ *for any* $\rho_r \in [0, \rho_{\hat{r}})$*, and* $y \succ_r x$ *for any* $\rho_r \in (\rho_{\hat{r}}, 2]$.

Proposition 2 implies a monotonic behavior with respect to the regret-weighting index. Intuitively, the smaller ρ_r is, the less weight the individual places on the regret-rejoice function associated with choosing x, which increases its likelihood of being chosen.

RDRT's explanatory power

We begin with a discussion of the aversion to exchange lottery tickets (Risen & Gilovich, 2007; Van de Ven & Zeelenberg, 2011), which is an example of the omission

[10] See Appendix A for a proof of this and other results in the paper.

bias. Suppose that there are N lottery tickets and that exactly one ticket is the winner (with probability 1/N). An individual holding one such ticket has been given the opportunity to keep it (act x) or exchange it with another one (act y). Under the original regret theory, both acts would result in the same (modified) utility. That is because holding on to the first ticket, or exchanging for another one, would entail the same outcomes in every state of the world — winning or not. Formally, the utility for *both* acts is equal to:

$$\frac{1}{N}[u(W)+\Phi(u(W)-U(L))]+\frac{N-1}{N}[u(L)+\Phi(u(L)-U(W))] \tag{7}$$

where W and L correspond to the outcomes of winning and losing the lottery, respectively. Let us now assume that the decision maker's reference act r, determined by the most recent belief regarding her expected (state-contingent) wealth, includes the ticket. Since $x_s = r_s$ for every state s, it follows that $\rho_r(x|y) = 0$. The utility of choosing x is thus:

$$\sum_{s\in S} p_s(u(x_s)+\rho_r(x|y)\Phi(u(x_s)-u(y_s)) = \frac{1}{N}u(\mathrm{W})+\frac{N-1}{N}u(\mathrm{L}) \tag{8}$$

Note that the utility of keeping the original ticket is the same as the standard EU case due to cancelling out of the regret-rejoice components. Exchanging her ticket with another one (act y), on the other hand, implies moving away from the reference point, which now puts full weight on the regret-rejoice function. There are now three relevant states of the world: the original ticket is the winner, the ticket switched to is the winner, or neither ticket is the winner. The utility associated with act y is given by:

$$\sum_{s\in S} p_s(u(y_s)+\rho_r(y|x)\Phi(u(y_s)-u(x_s)) =$$

$$\frac{1}{N}u(\mathrm{W})+\frac{N-1}{N}u(\mathrm{L})+\frac{2}{N}\Phi(u(\mathrm{W})-u(\mathrm{L}))+\frac{2}{N}\Phi(u(\mathrm{L})-u(\mathrm{W})) \tag{9}$$

Note that the overall utility in (9) has to be lower than the utility of keeping the original lottery ticket (Equation 8) as long as properties A1-A3 are satisfied. The model thus predicts that the decision maker will strictly prefer the "passive" action of keeping her chosen lottery ticket, even though the two lotteries are ex-ante identical. This result is generalized in the following corollary to Proposition 2.

Corollary 1 (Lottery Exchange Aversion) *Let x and y be two acts such that $x \sim_r y$ for $\rho_r = 1$, and $x_s \neq y_s$ for some state $s \in S$. Then, $x \succ y$ for any r such that $\rho_r(x|y) < 1$.*

We turn next to showing how RDRT can account for the common-ratio effect with juxtaposition.[11] The necessary condition here is that the reference point be different than the status quo of current wealth. To see this, assume first that the (constant) reference point r is the current wealth α. Consider two prospects x and y with $x = \{(\alpha+\beta, p); (\alpha, 1-p)\}$ (read: $\alpha+\beta$ with probability p and α with probability $1-p$) and $y = \{(\alpha+\gamma, \lambda p); (\alpha, 1-\lambda p)\}$, where $\gamma > \beta \geq 0$ are two monetary outcomes and $\lambda \in (0,1)$ represents the common ratio for any level of $p \in (0,1]$. The weighting index of x given alternative y for reference point r is:

$$\rho_r(x|y) = \frac{2}{1+\sqrt{\lambda}\frac{u(\alpha+\gamma)-u(\alpha)}{u(\alpha+\beta)-u(\alpha)}}$$

Note that in the case where $r = \alpha$, the regret-weighting index is invariant to p. Table 5 presents this choice problem using certain juxtapositions, where {s1, s2, s3} is a set of collectively exhaustive and mutually exclusive states of the world with probabilities $P(s_1) = \lambda p$, $P(s_2) = (1-\lambda)p$, and $P(s_3) = 1-p$. Note that the decision problems in Figure 1 are obtained here as a special case. Hence, with the regret-weighting index being independent of p, and since state s_3 generates no feeling of regret (or rejoicing), then for all of $p \in [0,1]$ the reference dependent model will consistently predict either $x \succ y$ or $y \succ x$. This prediction is in line with that of standard regret theory, yet it is inconsistent with the observed behavior of the common-ratio effect with juxtaposition (Loomes & Sugden, 1998; Sugden, 2003).

This need not be the case, however, if the reference point is such that $r \neq \alpha$. If, for instance, r is a constant reference act equal to $\alpha+\beta$, then the regret-weighting index becomes:

$$\rho_r(x|y) = \frac{2}{1+\sqrt{\frac{\lambda p}{1-p}\left[\frac{u(\alpha+\gamma)-u(\alpha+\beta)}{u(\alpha)-u(\alpha+\beta)}\right]^2+\frac{1-\lambda p}{1-p}}}$$

which is now a function of p. When $p = 1$, the decision maker perceives x as the safe option and we have $\rho_r(x|y) = 0$, implying no regret or rejoicing from choosing x. As p approaches zero, on the other hand, the regret-weighting index $\rho_r(x|y)$ approaches $\rho_r(y|x)$, and the preference under LS's regret theory is obtained. This can result in a

[11] In Proposition A.1 in the Appendix we provide a formal result regarding the common-ratio effect with independent prospects. In sum, the result is obtained under some additional constraints on the reference point.

choice reversal, choosing x for "high" levels of p (above a certain threshold) and y for "low" levels of p, which can explain the common-ratio effect even with juxtaposition as the method of presentation.

Proposition 3 (Common-ratio effect with juxtaposition) *Let* $\{s_1, s_2, s_3\}$ *be a set of collectively exhaustive and mutually exclusive states of the world with probabilities* $P(s_1) = \lambda p$, $P(s_2) = (1 - \lambda)p$, $P(s_3) = 1 - p$, *where* $\lambda \in (0, 1)$ *and* $p \in (0, 1]$. *Let* x *and* y *be two acts such that* $x = \{(\alpha + \beta, P(s_1 \cup s_2)); (\alpha, P(s_3))\}$ *and* $y = \{(\alpha + \gamma, P(s_1)); (\alpha, P(s_2 \cup s_3))\}$, *where* α *is the current wealth and* $\gamma > \beta \geq 0$ *are two outcomes. Suppose* $y \succ x$ *under EU theory. For any constant reference point* $r \neq \alpha$ *such that* $u(r) < \frac{u(\beta)^2 - \lambda}{2(u(\beta) - \lambda)}$, *if there exists* $\hat{p}$ *such that* $x \sim_r y$, *then* $x \succ_r y$ *for* $p > \hat{p}$, *and* $y \succ_r x$ *for* $p < \hat{p}$.

Lastly, we will examine the implications of the reference dependent regret model for the case of two equiprobable outcomes. Recall that, in this case, the standard regret model reaches the same predictions as EU. Generally, this need not be the case in RDRT. Assume, for instance, an individual with a linear utility function, $u(x) = x$ and a (constant) reference point equal to her current wealth level α. Suppose the decision maker is offered a 50/50 chance to win or lose \$100. The EU certainty equivalent of this prospect is her current wealth, so EU theory would predict indifference between rejecting (option x) and accepting the offer (option y). Proposition 1 implies that regret theory would reach a similar conclusion. Since the standard regret model is obtained as a special case of the reference dependent regret model for $\rho_r(x|y) = \rho_r(y|x) = 1$, the same prediction of indifference between x and y can also be obtained under RDRT. However, in this example we are given that $r = x$, which implies that rejecting the offer is perceived as a "passive" choice (that maintains the status quo, i.e., $\rho_r(x|y) = 0$). Proposition 2, then, implies that the indifference relation breaks and we obtain a strict preference to reject the offer: $x \succ_r y$. Choice behavior for the general equiprobable-outcomes case under RDRT is captured by the following proposition:

Proposition 4 (Regret premium with equiprobable outcomes) *Assume that properties A1-A3 hold. Let* x *be an arbitrary act with two equiprobable consequences* x_1 *and* x_2. *Let* z *be a constant act such that* $z_1 = z_2 = z$ *and* $u(z) = \frac{1}{2}(u(x_1) + u(x_2))$. *Then for any constant reference point* r, $z \succ_r x$ *and* $CE_{EU} - CE_{RDRT} > 0$ *(i.e., positive regret premium).*

Proposition 4 states that if an outcome z is the EU certainty equivalent of an equiprobable two-consequences act, then under RDRT the safer alternative ought to be chosen given a constant reference point r. This essentially indicates a higher degree of risk aversion compared to the standard EU model, which is reflected in a larger risk premium. As a result, the decision maker is willing to pay an extra amount to avoid the larger regret induced by reference dependence, therefore the regret premium under our model is nonzero in contrast to Proposition 1.

Furthermore, if the differences between x_1, x_2 and z remain constant, then as the distance between x and r increases, $\rho_r(x|z)$ approaches $\rho_r(z|x)$, reducing RDRT to the standard version of regret theory, which, for the two equiprobable consequences case, coincides with EU. The following example illustrates this idea. Assume an agent with a reference point set as her current (constant) wealth. She might decide to reject a 50% chance to gain \$25 (\$0 otherwise) for the opportunity to choose a safe option of \$10 with certainty. This kind of first-order risk aversion can be captured by RDRT even for a concave choiceless utility function $u(\cdot)$ that is characterized by a "mild" Arrow-Pratt index of relative risk aversion (such as $u(x) = log(x)$, for example). However, behavior might get closer to EU's predictions as the amounts move away from her reference point. Namely, a 50/50 chance of winning \$10,025 or \$10,000 might seem now more attractive than the safe option of gaining \$10,010 with certainty.

Crucially, the fact that Proposition 4 holds for any reference point implies that RDRT is not subject to Rabin's critique as, unlike the standard regret theory, it departs from EU theory even for the two equiprobable consequences case. Hence, rejecting a 50/50 chance to gain \$11 or lose \$10 need not involve a very high curvature of the Bernoulli utility function $u(\cdot)$, nor does it require any limitations on the sensitivity $\Phi(\cdot)$ to gains and losses.

Relation to the Literature

In this section, we compare our model with other leading theories of decision under risk. We start by discussing extensions of LS's regret theory. Later, we turn to analyzing models based upon reference dependence. Naturally, many prominent models are left out of the discussion simply for not falling under one of these categories (e.g., the PRAM model in Loomes 2010). We restrict attention to those two types of models to maintain the focus on the main underlying forces of our proposed theory, namely regret aversion and reference dependence.

Other extensions of regret theory

Diecidue and Somasundaram (2017) provide an (axiomatic) behavioral foundation for the canonical form of equation (1). Their five necessary and sufficient conditions can be extended to RDRT as follows:

(i): For any r, $\succ_r$ is complete if $x \succ_r y$ or $y \succ_r x$ for any two acts x and y.

(ii): For any r, $\succ_r$ is continuous if the sets $\{x : x \succeq_r y\}$ and $\{x : y \succeq_r x\}$ are closed subsets for every y.

(iii): Strong monotonicity holds if for any r and any acts x and y, $x \succ_r y$ whenever $x_s \geq y_s$ for all states s and $x_s > y_s$ for some state s.

(iv): Dominance-transitivity holds if for any r and any acts x, y and z, $x \succ_r y$ and $y \succeq_r z$ implies $x \succ_r z$; similarly, $x \succeq_r y$ and $y \succ_r z$ implies $x \succ_r z$.

(v): Trade-off consistency holds if for any r and any acts x, y, z and w, and four outcomes α, β, γ and δ, such that $\alpha_s x \sim_r \beta_s y$, $\gamma_s x \sim_r \delta_s y$, and $\alpha_{s'} z \sim_r \beta_{s'} w$, then $\gamma_{s'} z \sim_r \delta_{s'} w$, where $\alpha_s x$ denotes an act identical to x except in state s, in which the outcome x_s is replaced by the outcome α.

RDRT satisfies all of the axioms except trade-off consistency (v). Trade-off consistency says that if at state s, receiving outcome α instead of outcome β is an equally good improvement as receiving γ instead of δ, then the same property should also hold for state s'. An implication of the axiom is that given two acts x and y, and two outcomes α and β, the indifference between $\alpha_s x$ and $\alpha_s y$ is a necessary and sufficient condition for the indifference between $\beta_s x$ and $\beta_s y$. Notice that the axiom imposes a strong notion of symmetry as every state-wide trade-off should be consistent and independent of the acts under consideration. In our model, the trade-off necessarily depends on the states and acts because the existence of a reference point leads to a change in the relative weighting index.

The development of different sets of axioms is reflected in the functional form of $Q(\xi)$. By allowing for a weaker notion of transitivity, regret theory breaks away from the linearity of $Q(\xi)$ required by EU. By relaxing the symmetry embedded in the trade-off consistency, our model extends the *skew symmetric* convex function $Q(\xi)$ to the regret dependent function $Q_r(\xi)$, which grants us the power to further accommodate anomalous behavior from the perspective of expected utility and anticipated regret.

Quiggin (1994) characterizes the functional form for regret theory with general choice sets where the regret of an action depends only on the actual outcome and the best attainable alternative in each state of the world. Gabillon (2020) further generalizes Quiggin (1994) to accommodate any feedback structure. While a general choice set and value of information are important considerations in the development of regret theory, we focus here on the canonical binary alternatives, no feedback setting in strict analogy to Loomes and Sugden (1982). The asymmetric feeling of regret induced by the reference point, which is central to RDRT is, to the best of our knowledge, novel to the literature.

Other reference-dependent models

An alternative class of models rely on the notion of a reference point to explain risky decision making that systematically deviates from EU theory. In this section we discuss three popular such models: Cumulative Prospect Theory (Tversky & Kahneman, 1992, henceforth CPT), Reference-Dependent Risk Attitude (Kőszegi & Rabin, 2007) and Reference-Dependent Subjective Expected Utility (Sugden, 2003). In particular, we will examine each model's predictive power with regards to the motivating examples discussed earlier.[12]

We first compare our model with cumulative prospect theory (CPT). Following a large number of experiments showing patterns of risky choice behavior that contradicts the predictions of EU theory, Kahneman and Tversky (1979) introduced the prospect theory model, which was later improved to become the more mathematically sound model known as CPT. This model was designed to account for anomalous choice patterns using prospects (with probability distribution defined over outcomes) as the elements of choice. A key principle of CPT involves a reference point, such that every outcome is properly evaluated as either a gain or a loss. Generally, Tversky and Kahneman (1992) assume that the reference point is equal to the "current asset position".

With an inverse S-shaped weighting function, CPT can predict a variety of "irrational" choice behaviors, such as the common-ratio effect. That said, since probability distributions are defined over outcomes rather than states of the world, CPT does not distinguish between the common-ratio effect with or without juxtaposition as the method of presentation; it predicts them irrespective of the correlational structure of different prospects (e.g. whether prospects are independent or not). Similarly, in the

[12] Formal derivations of these results can be found in Appendix B.

absence of a state-dependent reference point, CPT fails to predict the reluctance to exchange lottery ticket (or the omission bias more generally).

On the other hand, CPT is able to accommodate the reflection effect due to the distinctive curvature of the value function, which changes from convexity to concavity when crossing the reference point from the negative to the positive domain. Furthermore, the loss aversion property allows for first-order risk aversion and is, therefore, immune to Rabin's critique. That is, turning down a 50/50 win \$11 or lose \$10 gamble entails no restrictions on the declining rate of sensitivity to gains.

Another influential loss-aversion model was introduced by Kőszegi and Rabin (2007) — henceforth KR. Theirs is a prospect-based model that allows for stochastic reference points. A distinctive feature of the KR model is that the reference point is determined endogenously as part of the decision process, particularly when the individual can correctly foresee the choice sets and plan accordingly. For situations where the individual cannot commit to a choice until "shortly" before the resolution of uncertainty, KR define an equilibrium concept and call it preferred personal equilibrium (PPE). In essence, when considering a plan of action, the reference point is set so as to match it. If the individual expects to follow through with this plan, such that no other choice from the expected choice set yields higher utility (taking that reference point as given), then this plan is defined by KR as unacclimating personal equilibrium (UPE). Generally, there could be multiple choices that satisfy the UPE definition. PPE is thus defined as the UPE that yields the highest utility of all such equilibria.

In terms of lottery exchange aversion, the KR model makes the same prediction as EU theory, i.e., an indifference between keeping the original ticket or exchanging it. Similar to prospect theory, this result is obtained, again, due to the absence of a state-dependent reference point. The model is also able to predict the common-ratio effect for the case with a nonlinear probability weighting function, though the result does not obtain with linear decision weights. Moreover, having a loss-gain utility with the same functional form as the prospect theory value function — including the loss aversion property —, the model can account for the reflection effect as well as for first-order risk aversion, which makes it immune to Rabin's critique.

The third model we consider is the Satisfaction-Change Subjective Expected Utility model of Sugden (2003). The model is based on Savage's axioms of subjective EU theory (Savage, 1972), with modifications to accommodate the existence of a reference point. Under this framework, preferences are defined over a set of state-contingent acts with respect to some reference act, where an act is a function from states of the world to consequences. Note that unlike the above two loss aversion models, Sugden's model is state-based rather than prospect-based. Also, according to Sugden: "A decision problem can be described by a reference act (interpreted as the agent's status quo position) and an opportunity set of acts (the set of options from which the agent must choose), of which the reference act is one element" (Sugden, 2003, p. 175).

Sugden's axioms give rise to a unique representation of reference-dependent SEU. He then chooses to focus on a special case of such representation which he calls the satisfaction-change SEU representation (henceforth, SCSEU). If the reference point of an individual is a constant act (e.g., her current, constant wealth), then Sugden's model both fails to predict the reflection effect and is exposed to Rabin's critique, as it coincides with the standard SEU. For the same reason, it cannot account for the common-ratio effect, with or without juxtaposition, despite being able to distinguish between different levels of correlations among state-contingent acts. Note, however, that SCSEU is able to capture the phenomenon of lottery exchange aversion due to the model's state dependent framework. From a technical standpoint, this result is obtained in a similar fashion to that presented in RDRT. Nevertheless, the two models clearly differ in the underlying psychological assumptions that drive such behavior, namely, regret versus gain-loss evaluation relative to the reference point.

Discussion & Conclusion

The original regret theory, introduced by Loomes and Sugden (1982), is a modification of the EU model that can rationalize a number of risk-taking behaviors that deviate from the standard EU model. It does so by recognizing the important role of regret (and rejoicing) as an underlying psychological force influencing choices under risk. However, LS's model is not able to accommodate a number of empirical regularities where regret is likely to be a main driver of behavior. In particular, RT fails to predict the often observed aversion to exchanging lottery tickets (Bar-Hillel & Neter, 1996; Kogler et al., 2013; Van de Ven & Zeelenberg, 2011), as well as various other examples of the omission bias. The theory also cannot account for the common-ratio effect when state-dependent acts are presented with juxtaposition, which aids the visualization of forgone alternatives. Moreover, in a subclass of choice problems with two equiprobable outcomes, predictions of regret theory coincide with those of EU theory, thus exposing the model to Rabin's critique.

In this paper we propose a reference dependent regret model (RDRT) as an extension of the original regret theory. Our model expands the range of predicted behaviors while retaining the notion of regret as an important driver of choice under uncertainty. The additional component added to this model is a state-contingent reference point that allows for asymmetric feelings of regret. In particular, a higher degree of regret is associated with more "active" choices, as measured by the weighted distance of a chosen act from the reference point relative to that of the alternative. As opposed to the original regret theory formulation, RDRT can account for regret-driven biases toward omission (e.g., the lottery-exchange aversion, preference toward inaction in vaccination decisions, etc.), and also disengage from EU theory (and, thus, from Rabin's critique) in the equiprobable-consequences case, thus allowing for first-order risk aversion. Furthermore, under certain conditions on the reference point (e.g., not being equal to current wealth), the model can also predict the correlated, state-contingent version of the common-ratio effect.

In addition, we conducted a direct comparison of our reference-dependent regret model with alternative extensions of regret theory (Diecidue & Somasundaram, 2017; Gabillon, 2020; Quiggin, 1994) and alternative models of reference-dependent preferences (Kahneman & Tversky, 1979; Kőszegi & Rabin, 2007; Sugden, 2003). Table 6 provides each model's predictions for a number of relevant cases discussed in the paper. It also contains the results for other key predictions of Loomes and Sugden (1982) — i.e., common-ratio effect with independent gambles and simultaneous gambling and insurance.[13]

Unlike other extensions of regret theory, our model highlights the asymmetric feeling of regret and moves away from LS's model by relaxing the axiom of trade-off consistency. Alternative reference-dependent models can explain part of our motivating examples, but they do so by relying on different psychological principles than regret to describe choice behavior under risk. Note that prospect theory and KR's model are both prospect-based models and, as such, are not well-suited to explain choices with correlated, state-contingent acts. In fact, as Table 6 shows, Sugden's theory and RDRT are the only models able to predict the lottery-exchange aversion (and, more broadly, to account for similar cases of the omission bias). The former, however, follows the SEU axioms and cannot predict the common-ratio effect regardless of the correlation structure of the problem. Moreover, Sugden (2003)'s model is also exposed to Rabin's

[13] See Appendix A for formal derivations of these results.

critique. We conclude that, with regards to our motivating examples, RDRT performs comparably better than the other generalized EU models discussed here. It does so by employing the notion of regret which plays (or is expected to play) an important role in the decision-making process.[14]

Another possible application of the model, which has not been explored here, is demand for insurance. Monetary outcomes from insuring a risk are essentially state dependent, which is consistent with RDRT. Furthermore, attitudes toward modest-scale risks — as reflected by preferences for policies with low deductibles, demand for cellular-phone insurance, extended warranties, etc. — suggest a higher level of risk aversion than the standard EU model would predict (Sydnor, 2006, 2010). This could be explained by RDRT assuming that an offer for insurance that the individual receives roughly corresponds to, or at least shapes her, reference point. If, for example, the decision maker considers an online offer for insurance where a default (low) deductible is already marked, she might incorporate the default state into her reference point. In which case, choosing the offer closest to the reference point (e.g., the low deductible offer), the more "passive" choice, yields a lower level of regret and, therefore, may lead to high risk-averse behavior, as predicted by our model. Future research on this subject could clearly shed more light and help determine the validity of this argument.

[14] Model complexity is another critical dimension for model evaluation (Pitt & Myung, 2002). Adding parameters to a model often extends the model's explanatory power, but also adds to model complexity. It would be interesting to see how these models differ in model complexity, on top of their explanatory power. We leave that to future extensions.

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Table 1

A Decision Problem with State-Contingent Outcomes

Act	$s_1(p = 0.25)$	$s_2(p = 0.25)$	$s_3(p = 0.25)$	$s_4(p = 0.25)$
x	$30	$20	$10	$0
y	$20	$10	$0	$30

x and y are acts with state-contingent consequences. Both acts result in the same probability distribution over outcomes, namely, 25% chance of each outcome $0, $10, $20, and $30. For this reason, any decision theory formulated in terms of prospects (i.e., probability distributions over final outcomes) would predict indifference between the two acts. Under regret theory, however, it is possible for a decision maker to prefer y over x, provided her regret aversion *is large enough.*

Table 2

An Illustration of Intransitive Preferences within Regret Theory

Act	$s_1(p = 0.25)$	$s_2(p = 0.25)$	$s_3(p = 0.25)$	$s_4(p = 0.25)$
x	\$30	\$20	\$10	\$0
y	\$20	\$10	\$0	\$30
z	\$10	\$0	\$30	\$20
w	\$0	\$30	\$20	\$10

Under the simplifying assumption that $u(x) = x$, regret theory would generate a cycle of preferences that violates transitivity for the four acts depicted. Specifically, we would have that $y \succ x$, $x \succ w$, $w \succ z$, and $z \succ y$. Note also that the same predictions could be arrived at with a strictly concave utility function, as long as $Q(\cdot)$ is convex enough.

Table 3

Regret Theory, Correlated Prospects, and the Common-Ratio Effect

Panel A

Act	$s_1(p = 0.20)$	$s_2(p = 0.80)$
x	\$3000	\$3000
y	\$0	\$4000

Panel B

Act	$s_1(p = 0.75)$	$s_2(p = 0.05)$	$s_2(p = 0.20)$
x'	\$0	\$3000	\$3000
y'	\$0	\$0	\$4000

Table 4

Example Decision Problems from Loomes & Sugden (1998)

Decision Problem 1

Act	$s_1(p = 0.40)$	$s_2(p = 0.60)$
x	$10	$10
y	$0	$30

Decision Problem 2

Act	$s_1(p = 0.75)$	$s_2(p = 0.10)$	$s_3(p = 0.15)$
x'	$0	$10	$10
y'	$0	$0	$30

Decision Problem 3

Act	$s_1(p = 0.10)$	$s_2(p = 0.30)$	$s_3(p = 0.60)$
z	$0	$10	$10
w	$0	$0	$30

Decision Problem 4

Act	$s_1(p = 0.70)$	$s_2(p = 0.10)$	$s_3(p = 0.20)$
z'	$0	$10	$10
w'	$0	$0	$30

Table 5

A Decision Problem with State-Contingent Outcomes

Act	s_1	s_2	s_3
x	$\alpha + \beta$	$\alpha + \beta$	α
y	$\alpha + \gamma$	α	α

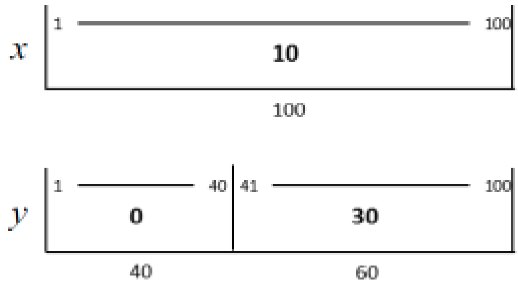
Table 6

A Summary of Comparison across Different Models Across Selected Domains

	Expected Utility	Regret Theory	Prospect Theory	KR (2007)	Sugden (2003)	RDRT (this paper)
Lottery Exchange (Omission Bias)	N	N	N	N	Y	Y
Common Ratio Effect with Juxtaposition	N	N	Y	Y	N	Y*
Common Ratio Effect with Independent Prospects	N	Y	Y	Y	N	Y
Simultaneous Gambling and Insurance	N	Y*	Y	Y	N	Y*
Reflection Effect	N	Y*	Y	Y	N	Y*
First-Order Risk Aversion or Rabin's Critique (equiprobable case)	N	N	Y	Y	N	Y

Note: Y indicates the model's ability to predict the relevant behavior. Y indicates the result holds with restrictions on either the utility function or the reference point. N indicates the model's inability to predict the relevant behavior. Formal derivations of these results are presented either in the main text or are derived formally in Appendix A.*

Figure 1. Visual Presentation of Binary Choices Between Correlated Acts in Loomes and Sugden (1998)



Appendix A

Proofs

Proof of Lemma 1

By A3, for all $\xi > 0$, $\Phi'(\xi) < \Phi'(-\xi)$. Integrating both sides, we get $\int_{\xi_2}^{\xi_1} \Phi'(\xi) d\xi < \int_{\xi_2}^{\xi_1} \Phi'(-\xi) d\xi$. That is, $\Phi(\xi_1) + \Phi(-\xi_1) < \Phi(\xi_2) + \Phi(-\xi_2)$ for any $\xi_1 > \xi_2$. ■

Proof of Proposition 1

By the definition of $Q(\cdot)$, we have

$$\begin{aligned}
\frac{1}{2}\sum_{i=1}^{2} Q(u(z) - u(x_i)) =& \frac{1}{2}\sum_{i=1}^{2} [u(z) - u(x_i) + \Phi(u(z) - u(x_i)) - \Phi(u(x_i) - u(z))] \\
=& \frac{1}{2} Q(\frac{u(x_2) - u(x_1)}{2}) + \frac{1}{2} Q(\frac{u(x_1) - u(x_2)}{2}), \\
=& 0,
\end{aligned}$$

where the second inequality comes from the assumption $u(z) = \frac{u(x_1)+u(x_2)}{2}$, and the last inequality comes from the fact that $Q(-\xi) = -Q(\xi)$ for all ξ.

Now if $u(z) > \frac{u(x_1)+u(x_2)}{2}$, then $u(z) - u(x_1) > \frac{u(x_2)-u(x_1)}{2}$ and $u(z) - u(x_2) > \frac{u(x_1)-u(x_2)}{2}$. By the properties of $Q(\cdot)$, we have

$$\frac{1}{2}\sum_{i=1}^{2} Q(u(z) - u(x_i) > \frac{1}{2} Q(\frac{u(x_2) - u(x_1)}{2}) + \frac{1}{2} Q(\frac{u(x_1) - u(x_2)}{2}) = 0.$$

Similarly, the reverse inequality is obtained for $u(z) > \frac{u(x_1)+u(x_2)}{2}$. This completes the proof. ■

Proof of Proposition 2

Substituting $\rho_r(y|x) = 2 - \rho_r(x|y)$ into $Q_r(\cdot)$ and rearranging, we get

$$\begin{aligned}
\sum_{s\in S} p_s Q_r(u(x_s) - u(y_s)) =& \sum_{s\in S} p_s \left[u(x_s) - u(y_s) - 2\Phi(u(y_s) - u(x_s))\right] \\
&+ \rho_r(x|y) \sum_{s\in S} \left[\Phi(u(x_s) - u(y_s)) + \Phi(u(y_s) - u(x_s))\right].
\end{aligned}$$

From lemma 1, we know that for all $s \in S$, $\Phi(u(x_s) - u(y_s)) + \Phi(u(y_s) - u(x_s)) \leq 0$. Moreover, since $x \neq y$, the inequality is strict. Thus the expected value of the $Q_r(\cdot)$ is strictly decreasing in ρ_r.

By assumption, there exists $\hat{\rho}_r$ such that $x \sim y$, equivalently

$$\sum_{s \in S} p_s \left[u(x_s) + \hat{\rho}_r(x|y)\Phi(u(x_s) - u(y_s)) \right] = \sum_{s \in S} p_s \left[u(y_s) + \hat{\rho}_r(y|x)\Phi(u(y_s) - u(x_s)) \right].$$

Therefore $x \succ y$ for any $\rho_r \in [0, \hat{\rho}_r)$, and $x \prec y$ for any $\rho_r \in (\hat{\rho}_r, 1]$. ■

Proof of Proposition 3

Rewriting the weighting index as

$$\rho_r(x|y) = \frac{2}{1 + \frac{\tilde{\rho}(y,r)}{\tilde{\rho}(x,r)}},$$

which is monotonically decreasing in the ratio of the distance $\frac{\tilde{\rho}(y,r)}{\tilde{\rho}(x,r)}$.

Without loss of generality, we normalize $u(\alpha) = 0$, and $u(\alpha + \gamma) = 1$. Deonte $u(\alpha + \beta) = u(\beta)$. Rearranging the ratio of distance, we have

$$\frac{\tilde{\rho}(y,r)}{\tilde{\rho}(x,r)} = \sqrt{\frac{\lambda p(1 - 2u(r)) + u(r)^2}{p(u(\beta)^2 - 2u(r)u(\beta)) + u(r)^2}}.$$

The ratio is monotonically increasing in the term inside the square root. Taking the partial derivative of that w.r.t. p, we get

$$\frac{u(r)^2 \left[\lambda(1 - 2u(r)) - (u(\beta)^2 - 2u(r)u(\beta)) \right]}{\left[p(u(\beta)^2 - 2u(r)u(\beta)) + u(r)^2 \right]^2}.$$

Since $u(\beta) < \lambda$ given the assumption that $y \succ x$ under EU, given $u(r) < \frac{u(\beta)^2 - \lambda}{2(u(\beta) - \lambda)}$, we have

$$2u(r)(u(\beta) - \lambda) > u(\beta)^2 - \lambda,$$

which implies the above partial derivative is larger zero. Therefore, $\rho_r(x|y)$ is monotonically decreasing in p.

It follows that for $p > \hat{p}$,

$$\begin{aligned} &\lambda p Q_r(u(\beta) - 1) + (1 - \lambda) p Q_r(u(\beta)) + (1 - p) Q_r(0) \\ \geq\ &\lambda \hat{p} Q_r(u(\beta) - 1) + (1 - \lambda) \hat{p} Q_r(u(\beta)) + (1 - \hat{p}) Q_r(0) = 0, \end{aligned}$$

where the second equality comes from the fact that $x \sim_r y$ at $p = \hat{p}$. Therefore, $x \succ_r y$ for $p > \hat{p}$. ■

Proof of Proposition 4

The weighting index is

$$\rho_r(z|x) = \frac{2}{1 + \sqrt{2\frac{(u(x_1)-u(r)))^2+(u(x_2)-u(r))^2}{[u(x_1)-u(r)+u(x_2)-u(r)]^2}}}.$$

Since $x_1 \neq x_2$, it follows that

$$\frac{(u(x_1)-u(r)))^2+(u(x_2)-u(r))^2}{[u(x_1)-u(r)+u(x_2)-u(r)]^2} > 1.$$

Consequently, $\rho_r(z|x) < 1$. By Proposition 1, $z \sim x$ if $\rho_r = 1$. Therefore, $z \succ_r x$ by Proposition 2. ■

Proposition A.1 (Common-ratio effect with independent prospects) *Let x and y be two independent prospects such that $x = \{(\alpha+\beta, p); (\alpha, 1-p)\}$ and $y = \{(\alpha+\gamma, \lambda p); (\alpha, 1-\lambda p)\}$, where α is the current wealth, $\gamma > \beta \geq 0$ are two outcomes, $\lambda \in (0,1)$ and $p \in (0,1]$. Suppose $y \succ x$ under EU theory. For any constant reference point r, if there exists $\hat{p}$ such that $x \sim_r y$, then $x \succ_r y$ for $p > \hat{p}$, and $y \succ_r x$ for $p < \hat{p}$.*

Proof. Without loss of generality, we normalize $u(\alpha) = 0$, and $u(\alpha+\gamma) = 1$. Deonte $u(\alpha+\beta) = u(\beta)$. $x \succ_r y$ if and only if

$$\lambda p^2 Q_r(u(\beta)-1) + p(1-\lambda p)Q_r(u(\beta)) + (1-p)\lambda p Q_r(-1) \geq 0.$$

Rearranging the terms, we get

$$p\{Q_r(u(\beta)) + \lambda Q_r(-1) - \lambda p[Q_r(u(\beta)) + Q_r(-1) - Q_r(u(\beta)-1)]\} \geq 0.$$

It follows from A2 that $Q_r(u(\beta)) + Q_r(-1) \leq Q_r(u(\beta)-1)$. Therefore, by intermediate value theorem, there exists a $\hat{p}$ such that if $p > \hat{p}$, then $x \succ_r y$, and if $p < \hat{p}$, $y \succ_r x$. ■

Proposition A.2 (Simultaneous Gambling and Insurance) *Let x and y be two independent prospects (that offer an actuarially fair gamble) such that $x = \{(\alpha, 1)\}$ and $y = \{(\alpha+\beta, p); (\alpha - \frac{p\beta}{1-p}, 1-p)\}$, where α is the current wealth, $\beta \geq 0$ is an outcome, and $p \in (0,1]$. Suppose $u(\cdot)$ is linear (as in LS). For any constant reference point r, if there exists $\hat{p}$ such that $x \sim_r y$, then $x \succ_r y$ for $p > \hat{p}$, and $y \succ_r x$ for $p < \hat{p}$.*

Proof. Without loss of generality, we normalize $u(\alpha) = 0$, and $u(\alpha+\gamma) = 1$. Deonte $u(\alpha+\beta) = u(\beta)$. $x \succ_r y$ if and only if

$$pQ_r(-\beta) + (1-p)Q_r\left(\frac{p\beta}{1-p}\right) \geq 0.$$

Since $Q_r(\beta) + Q_r(-\beta) = [\rho_r(x|y) - \rho_r(y|x)][\Phi(\beta) + \Phi(-\beta)]$, the above inequality is equivalent to

$$[\rho_r(x|y) - \rho_r(y|x)][\Phi(\beta) + \Phi(-\beta)] + \frac{1-p}{p} Q_r(\frac{p\beta}{1-p}) \geq Q_r(\beta).$$

Equivalently,

$$2(\rho_r - 1)[\Phi(-\beta) + \Phi(\beta)] + \left[\frac{1-p}{p} Q_r(\frac{p\beta}{1-p}) - Q_r(\beta)\right] \geq 0.$$

By A2, $\frac{1-p}{p} Q_r(\frac{p\beta}{1-p}) - Q_r(\beta)$ is increasing in p and greater than zero if $p > 1/2$. By A3, $\Phi(-\beta) + \Phi(\beta) < 0$. It remains to show that the weighting index ρ_r is decreasing in p. Equivalently, the ratio of distance

$$\begin{aligned} \frac{\tilde{\rho}(y,r)}{\tilde{\rho}(x,r)} &= \sqrt{\frac{p(\beta - r)^2 + (1-p)(\frac{p}{1-p}\beta + r)^2}{r^2}} \\ &= \sqrt{\frac{p\beta^2}{(1-p)r^2}} \end{aligned}$$

is increasing in p. ■

Proposition A.3 (Reflection Effect) *Let x and y be two independent prospects such that $x = \{(\beta, p_1); (0, 1-p_1)\}$ and $y = \{(\gamma, p_2); (0, 1-p_2)\}$, where $\gamma \geq \beta \geq 0$ are two outcomes, and $p_1, p_2 \in (0, 1]$. Denote their "reflections" as $x' = \{(-\beta, p_1); (0, 1-p_1)\}$ and $y' = \{(-\gamma, p_2); (0, 1-p_2)\}$. Suppose $u(\cdot)$ is linear (as in LS). For any constant reference point r such that $r \leq \frac{p_2\gamma^2 - p_1\beta^2}{2(p_2\gamma - p_1\beta)}$, then $x \succ_r y$ implies $y' \succ_r x'$.*

Proof. $x \succ_r y$ if and only if

$$p_1 Q_r(\beta) - p_2 Q_r(\gamma) - p_1 p_2 [Q_r(\beta) - Q_r(\beta - \gamma) = Q_r(\gamma)] \geq 0.$$

$r \leq \frac{p_2\gamma^2 - p_1\beta^2}{2(p_2\gamma - p_1\beta)}$ if and only if $\rho_r \leq 1$. Combining with A3, it follows that $p_2(1-p_1)[\rho_r(y|x) - \rho_r(x|y)][\Phi(\gamma) + \Phi(-\gamma)]$ is negative. Since $Q_r(-\gamma) + Q_r(\gamma) = [\rho_r(y|x) - \rho_r(x|y)][\Phi(\gamma) + \Phi(-\gamma)]$, the first equation can be rewritten as

$$p_1 p_2 Q_r(\beta - \gamma) + p_1(1-p_2) Q_r(\beta) + p_2(1-p_1) Q_r(-gamma) \geq 0.$$

which is the necessary and sufficient condition for $y' \succ_r x'$. ■

Appendix B

Mathematics of Other Reference-Dependent Models

CPT

In CPT, the monetary outcomes of each prospect are ordered ascendingly, where a neutral outcome (e.g., a reference point) is denoted by $x_0 = 0$, and gains and losses are denoted by positive and negative numbers, respectively. Then, every prospect, f, is evaluated according to the following functional form:

$$V(f) = \sum_{-m}^{0} \pi_i^- v(x_i) + \sum_{0}^{n} \pi_i^+ v(x_i), \tag{10}$$

where m and n represent the number of negative and positive consequences respectively, such that outcomes are ranked as:
$x_{-m} < x_{-m+1} < ... < x_{-1} < x_0 = 0 < x_1 < ... < x_{n-1} < x_n$, v is a value function that encodes the value of every outcome, π^- is the decision weight associated with losses, and π^+ is associated with gains.

The value function, v, is assumed to be concave for positive outcomes and convex for negatives, with $v(0) = 0$. Moreover, it is characterized by a steeper slope for losses than gains in order to reflect the loss aversion principle. The decision weights π^- and π^+ are defined by:

$$\pi_i^- = w^-[Pr(x \leq x_i) - Pr(x \leq x_{i-1})], -m+1 \leq i \leq 0,$$

$$\pi_i^+ = w^+[Pr(x \geq x_i) - Pr(x \geq x_{i+1})], 0 \leq i \leq n-1,$$

with $\pi_{-m}^- = w^-[Pr(x_{-m})]$, and $\pi_n^+ = w^+[Pr(x_n)]$, where w^- and w^+ are strictly increasing weighting functions that assign weights to cumulative probabilities of losses and gains, respectively, with $w^-(0) = w^+(0) = 0$, and $w^-(1) = w^+(1) = 1$. Both w^- and w^+ were estimated by KT to fit as inverse S-shaped functions. This functional form implies overweighting of small probabilities and underweighting of moderate and high probabilities.

In KR, there are two functions, the "consumption utility"$u(x_s)$, and the loss-gain function $\mu(u(x_s) - u(r_s))$, where x_s is the level of wealth and r_s is the reference wealth. $u(x_s)$ is in fact a basic utility of wealth, similar to that being applied in the standard EU of wealth theory. $\mu(u(x_s) - u(r_s))$ is a gain-loss function that assigns real values to changes in the level of utility relative to some reference point. It has the same structure of the prospect-theory value function as discussed above (i.e., diminishing sensitivity as well as a kink around the reference point that corresponds to the loss aversion assumption). According to the model, the total utility gained from choosing an alternative x given some reference point r, with p_s being the probability measure over the S possible wealth outcomes of x and q_t the probability measure over the T possible outcomes of r, is captured by the following expression:

$$U_r(x) = \sum_{s \in S} p_s u(x_s) + \sum_{s \in S} \sum_{t \in T} p_s q_t \mu(u(x_s) - u(r_t))) \tag{11}$$

Recall the scenario with the two doors, door x and door y, in which a million-dollar prize lies behind one of them. Suppose that door x has previously been chosen and a chance to switch doors has now been given. Then, according to the PPE concept, if the individual with some current level of wealth α, expects to keep door x (which makes door x as the reference point, denoted by $r = x$), she would gain a total utility of:

$$U_r(x) = \frac{1}{2}(u(\alpha + 10^6) + u(\alpha)) + \frac{1}{4}\mu(u(\alpha + 10^6) - u(\alpha)) + \frac{1}{4}\mu(u(\alpha) - u(\alpha + 10^6)).$$

Switching to door y yields:

$$U_r(y) = \frac{1}{2}(u(\alpha) + u(\alpha + 10^6)) + \frac{1}{4}\mu(u(\alpha) - u(\alpha + 10^6)) + \frac{1}{4}\mu(u(\alpha + 10^6) - u(\alpha)).$$

Clearly, if $U_r(x) \geq U_r(y)$ then keeping door x is a UPE by definition. Repeating the same exercise, with the expectation to switch doors this time, would make door y the reference point (denoted by $r = y$). However, since the KR model is a prospect-based model, then by symmetry $r = x$ and $r = y$ are equivalent, so that switching doors would also be a UPE. Furthermore, $U_r(x) = U_r(y)$ suggesting no unique PPE.

SCSEU is based on two functions, the satisfaction function $u(x_s)$, unique up to affine transformations, and the gain-loss evaluation function $\Phi(\cdot)$, which is an increasing function, unique up to a scale factor, with $\Phi(0) = 0$. $u(x_s)$ is a Bernoulli utility function over end states that encodes the level of satisfaction gained from state contingent outcomes, whereas $\Phi(\cdot)$ is a gain-loss function that measures the change in satisfaction relative to some reference act. Thus, an individual weakly prefers act x over y given a reference act r, if and only if

$$\sum_{s \in S} p_s \left[\Phi(u(x_s) - u(r_s)) - \Phi(u(y_s) - u(r_s))\right] \geq 0 \tag{12}$$

where p_s is a probability measure defined over the set of all possible states of the world. Note that when preferences are evaluated from the perspective of a constant reference act, with a fixed consequence $z_s = z$ for any s, the SCSEU representation reduces to Savage's SEU model, such that for any consequence x_s, Savage's utility function $U(\cdot)$ takes on the following form: $U(x) = \Phi(u(x_s) - u(z))$. In which case, Sugden's model cannot predict the reflection effect, or avoid being exposed to Rabin's critique, or explain the common ratio effect (with or without juxtaposition) as it coincides with the standard SEU. It can, however, account for the lottery exchange aversion. To see this, consider once again the decision problem of choosing between keeping door x or switching to door y, whereas behind exactly one door awaits a one-million-dollar prize. Given that the individual's status-quo reference act includes door x (it is part of the state contingent assets of the decision maker), she would (weakly) choose to keep door x over moving to door y, if and only if

$$\sum_{s \in S} [\Phi(u(x_s) - u(x_s)) - \Phi(u(y_s) - u(x_s))] \geq 0.$$

Since $\Phi(u(x_s) - u(x_s)) = 0$, the inequality can be rewritten as

$$\frac{1}{2}\Phi(u(\alpha) - u(\alpha + 10^6)) + \frac{1}{2}\Phi(u(\alpha + 10^6) - u(\alpha)) \leq 0, \tag{13}$$

where α is the individual's current level of wealth. This equation holds if $\Phi(\cdot)$ follows a property defined by Sugden as a (weak) zero point concavity. With this property the individual experiences (weak) exchange-averse preferences, a concept suggesting that "an agent is exchange-averse if, other things being equal, he prefers the status quo to other options in his opportunity set." (Sugden, 2003, p. 181), as can be seen in the lottery exchange example.[15]

[15] Notice that the a bias toward the status-quo need not be the same as the omission bias (Ritov & Baron, 1992).